Effects of Loss and Decoherence on a Simple Quantum Computer

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We investigate the impact of loss (amplitude damping) and decoherence (phase damping) on the performance of a simple quantum computer which solves the one-bit Deutsch problem. The components of this machine are beamsplitters and nonlinear optical Kerr cells, but errors primarily originate from the latter. We develop models to describe the effect of these errors on a quantum optical Fredkin gate. The results are used to analyze possible error correction strategies in a complete quantum computer. We find that errors due to loss can be avoided perfectly by appropriate design techniques, while decoherence can be partially dealt with using projective error correction.

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I. INTRODUCTION

Quantum computers utilize the non-locality of quantum physics to allow exponentially fast solutions to classical problems [1]. However, there is a catch. The fundamental element of a quantum computer is the quantum bit (qubit), which may be in a superposition state of zero and one. It is a very fragile state. Ideally, the quantum computer is a closed system, but in reality, when information leaks out the qubits collapse and errors are introduced into the calculation. Evaluation of the impact of this *decoherence* process is a key to understanding the feasibility of quantum computation [2,3,4,5].

In this paper we will investigate in detail the effect of decoherence on the quantum computer of Chuang and Yamamoto [6], which specifically implements the Deutsch-Josza solution to the one-bit oracle problem proposed by Deutsch [7]. The function of their machine is essentially to use an interference experiment to determine the class of a hidden function. There are only two possibilities, and in the absence of error the class is determined with certainty. The proposed realization uses an optical quantum computer with beam splitters and nonlinear Kerr medium (Figure 1). The function of each individual component is understood well from the study of quantum optics: the top and bottom pairs of single mode waveguides implement two qubits, the beamsplitter implements a $\sqrt{\text{NOT}}$ logic gate, and the Kerr medium implements a logic gate via cross-phase modulation between single photons.

Two important imperfections which lead to errors are energy loss and decoherence. The former occurs due to imperfect experimental implementation, such as facet reflections or absorption in waveguiding media. This loss of photons to the environment through scattering is a distributed process which is mathematically described as *amplitude damping*.

Decoherence is different; it is present even in cases in which energy loss is negligible. Idle photons decohere very little at room temperature as they interact weakly with their environment. However, in the optical quantum computer we are analyzing the situation is different – the atom-photon interaction that occurs when the photons interact with each other through a Kerr medium causes decoherence. In general, this interaction leaves correlations between atoms and photons thus leading to decoherence, since partial information about the photon's state is left behind in the atoms. This decoherence process is mathematically described as *phase damping* or phase randomization. The stochastic nature of decoherence likens it to a noise process and makes it a more insidious source of errors than loss; its time-scale may be shorter than for energy loss [8], and as a practical matter it is harder to avoid.

We will first calculate the general effect of loss and decoherence on an optical quantum Fredkin gate, which is a typical elementary operation for a quantum computer. Our models will then be used to analyze the effects on a complete system, the Chuang-Yamamoto quantum computer. Finally, we shall find these results useful in understanding how error correction can play a role in stabilizing quantum computations.

II. DECOHERENCE OF AN OPTICAL FREDKIN GATE

The apparatus used to construct an optical Fredkin gate [9,10] is shown in Figure 2. Let the usual creation and annihilation operators for the three modes be a, b, and c and their adjoints. In this language, the two beamsplitters (Figure 3) are described by the operators B and

$B^\dagger$, where

$$B = \exp\left[\frac{\pi}{4}\left(a^\dagger b - b^\dagger a\right)\right] . \tag{1}$$

Similarly, the cross-phase modulation component of the Kerr medium is usually described by

$$K = \exp\left[i\chi\, b^\dagger b\, c^\dagger c\right] , \tag{2}$$

with $\chi = \pi$ ideally. These definitions immediately give us the quantum Fredkin gate operator (with no damping and no decoherence),

$$F = B^\dagger\, K\, B\, . \tag{3}$$

The matrix elements of F relevant for the Chuang-Yamamoto quantum computer, to be discussed later, are

$$F\,|000\rangle = |000\rangle \tag{4}$$
$$F\,|100\rangle = |100\rangle \tag{5}$$
$$F\,|010\rangle = |010\rangle \tag{6}$$
$$F\,|101\rangle = |011\rangle \tag{7}$$
$$F\,|011\rangle = |101\rangle\, , \tag{8}$$

using the labeling $|\mathsf{abc}\rangle$. Note that $F^\dagger = F$.

Energy loss, i.e. the loss of a photon to the environment, can be described for a single qubit (in mode a) by the superscattering operator $\$^a_\gamma$

$$\$^a_\gamma \begin{bmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{bmatrix} = \begin{bmatrix} \rho_{00} + (1-e^{-\gamma})\rho_{11} & e^{-\gamma/2}\rho_{01} \\ e^{-\gamma/2}\rho_{10} & e^{-\gamma}\rho_{11} \end{bmatrix} . \tag{9}$$

A simple way of deriving this expression is by taking $1 - e^{-\gamma}$ as the probability for absorbing the qubit photon and creating an excitation in the unobserved environment. Mathematically, we may write the wavefuntion for the qubit + environment as

$$|10\rangle \rightarrow e^{-\gamma/2}|10\rangle + \sqrt{1-e^{-\gamma}}|01\rangle\, . \tag{10}$$

and arrive at the superscattering operator by summing over the environment, represented by the second label. Thus, $\$^a_\gamma$ describes amplitude damping due to coupling of a qubit to its environment.

The Kerr medium used in the quantum Fredkin gate is experimentally known to be lossy [11], and we may model this by inserting a loss mechanism in its arguments. The resulting Fredkin gate is described by the superscattering operator $\$_{F_\gamma} = B^\dagger \$^b_\gamma \$^c_\gamma K B$. (It can be shown that the physics does not change if the damping is distributed or placed before or after the Kerr medium.) It is clear that the worst affected states for the simple quantum computer are $|101\rangle$ and $|011\rangle$ for which

$$\$_{F_\gamma}\left[|101\rangle\langle 101|\right] = \frac{(1-e^{-\gamma})^2}{2}|000\rangle\langle 000| \tag{11}$$
$$+ \frac{e^{-\gamma}(1-e^{-\gamma})}{2}|001\rangle\langle 001|$$
$$+ \frac{(1-e^{-\gamma})}{4}|\phi_{01}\rangle\langle\phi_{01}| + \frac{e^{-\gamma}}{4}|\phi_{10}\rangle\langle\phi_{10}|\, , \tag{12}$$

where

$$\phi_{01} = (1+e^{-\gamma/2})\,|010\rangle + (1-e^{-\gamma/2})\,|100\rangle \tag{13}$$
$$\phi_{10} = (1+e^{-\gamma/2})\,|011\rangle + (1-e^{-\gamma/2})\,|101\rangle\, . \tag{14}$$

The first term in $\$_{F_\gamma}$ corresponds to the absorption of two photons in the Kerr medium. The second and third terms result from the absorption of one photon (from either mode b or c) and finally the last term correspond to no absorption at all in the Kerr medium. A similar result is obtained for $\$_{F_\gamma}(|011\rangle\langle 011|)$ by interchanging the first two qubits. Obviously energy loss induces errors by degrading the computer's state and transforming it into one with lower energy. However, we will see later how errors coming from energy loss can easily be detected and corrected.

Even if one is able to minimize energy losses, there will be another source of problems: damping of phase coherence which occurs as the photons go through the Kerr medium. In fact, the effective operator for the Kerr cell may be written as

$$K(\epsilon) = \exp\left[i\pi\, b^\dagger b\, c^\dagger c + i\eta\right] , \tag{15}$$

where η is an operator acting both on the photons and the atoms. This interaction produces correlations which, as the atoms are unobserved, generate decoherence. For the purpose of computing the trace over the environment we can treat η as if it were a simple function of a random variable ϵ. The source for randomness is, as described above, the atom–photon interaction taking place in the Kerr cell. We let

$$\eta = \epsilon\,(b^\dagger b + c^\dagger c)\, , \tag{16}$$

where ϵ is a random variable which zero mean (this will be justified in an explicit model later). Taking this into account, the relevant matrix elements of the quantum Fredkin gate with phase decoherence, F_λ, are

$$F_\lambda|000\rangle = |000\rangle \tag{17}$$
$$F_\lambda|100\rangle = \left[\frac{1+e^{i\epsilon}}{2}\right]|100\rangle + \left[\frac{1-e^{i\epsilon}}{2}\right]|010\rangle \tag{18}$$
$$F_\lambda|010\rangle = \left[\frac{1-e^{i\epsilon}}{2}\right]|100\rangle + \left[\frac{1+e^{i\epsilon}}{2}\right]|010\rangle \tag{19}$$
$$F_\lambda|101\rangle = \frac{e^{i\epsilon}}{2}\left[1-e^{i\epsilon}\right]|101\rangle + \frac{e^{i\epsilon}}{2}\left[1+e^{i\epsilon}\right]|011\rangle \tag{20}$$
$$F_\lambda|011\rangle = \frac{e^{i\epsilon}}{2}\left[1+e^{i\epsilon}\right]|101\rangle + \frac{e^{i\epsilon}}{2}\left[1-e^{i\epsilon}\right]|011\rangle\, , \tag{21}$$

Tracing over the environment formed by the atoms corresponds to averaging over the random variable ϵ. Assuming a Gaussian distribution, i.e. $\langle e^{i\epsilon}\rangle = e^{-\lambda}$, with $\lambda = \langle\epsilon^2\rangle$, one gets a superscattering operator, $\$_{F_\lambda}$, for the Fredkin gate. For example, its action upon the state $|011\rangle$ is

$$\$_{F_\lambda}\left[|101\rangle\langle 101|\right] = \frac{1+e^{-\lambda}}{2}|011\rangle\langle 011| + \frac{1-e^{-\lambda}}{2}|101\rangle\langle 101|\,, \tag{22}$$

Similarly, $\$_{F_\lambda}(|011\rangle\langle 011|)$ is obtained by switching the first two bits in the previous expression.

In a recent paper Boivin et al. [12] presented a model for a 1+1 dimensional Kerr cell with weak nonlinearity. The main conclusion of their analysis, based on the fact that the variable η in (15) can indeed be described by equation (16), is that non-linearity of the medium is unavoidably accompanied by phase noise of the field. Using their conclusions here implies the existence of a relationship between the amount of phase shift θ and the value of λ. In their example, for a coherent input state corresponding to a monochromatic pump at the carrier frequency, one gets that for a phase shift of $\theta = \pi$, the decoherence parameter $\lambda = \pi\Omega/I$, where Ω is the resonant frequency of the medium and I is the intensity of the pulse, in units of photon number per second. This would imply a rather large amount of decoherence per step of the quantum computer. It remains to be seen if their 1+1 dimensional model is reasonable.

III. ENERGY LOSS IN THE SIMPLE QUANTUM COMPUTER

Let us now apply our results to investigate the effect of energy loss and decoherence on the Chuang-Yamamoto computer. We begin by considering the effect of energy loss. The computer has two nontrivial settings, $k_1 = 0$ and $k_1 = 1$; when $k_1 = 0$, the unitary transform performed by the computer is

$$U_0 = B^\dagger{}_{cd}\$^{abe}_{F_\gamma} S_a F_{abe} B_{cd}\,, \tag{23}$$

where a,b,c,d denote the four optical modes used in the machine (mode e is always zero). We consider only the effect of loss in the second Fredkin gate by replacing the operator F_{abc} with the non-unitary superscattering operator $\$^{abc}_{F_\gamma}$ which describes a Fredkin gate with loss. The normal input to the machine is $|\mathsf{abcd}\rangle = |0101\rangle$, and e is the vacuum. Since the signal in modes c and d do not enter the Kerr medium when $k_1 = 0$, loss in the Kerr medium is irrelevant, and the answer either $|0101\rangle$ or $|0001\rangle$. The latter case is an error but it is easily detectable as only one photon is observed and thus must be incorrect (under proper operation, no photon is ever lost from the system).

On the other hand, when $k_1 k_0 = 10$, then the transform performed by the computer is

$$U_1 = B^\dagger{}_{cd}\$^{abc}_{F_\gamma} S_a F_{abc} B_{cd}\,, \tag{24}$$

such that for the first half of the apparatus, we have

$$\begin{aligned}
|\psi_0\rangle &= |0101\rangle \\
|\psi_1\rangle &= B_{cd}|\psi_0\rangle = \frac{1}{\sqrt{2}}\left[|0101\rangle + |0110\rangle\right] \\
|\psi_2\rangle &= F_{abc}|\psi_1\rangle = \frac{1}{\sqrt{2}}\left[|0101\rangle + |1010\rangle\right] \\
|\psi_3\rangle &= S|\psi_2\rangle = \frac{1}{\sqrt{2}}\left[|0101\rangle - |1010\rangle\right]
\end{aligned} \tag{25}$$

as the state before the second Fredkin gate. Using a density matrix description, we calculate

$$\rho_4 = \$^{abc}_{F_\gamma}\left[|\psi_3\rangle\langle\psi_3|\right] \tag{26}$$

$$\rho_5 = B^\dagger_{cd}\rho_4 B_{cd}\,, \tag{27}$$

where ρ_4 is the output of the second, lossy Fredkin gate, and ρ_5 is the final output. The diagonal elements of ρ_5 give us the final measurement result probabilities. Errors occur because of imperfect switching, as described by Eq.(12). Following [6], if the measurement of mode d is taken as the computation result, we find that the error probability is

$$P_{\rm NOEC} = \frac{1}{4}\left[1 + e^{-\gamma} - 2e^{-3\gamma/2}\right]\,. \tag{28}$$

However, the technique of using two modes to represent a single qubit (what we have termed the *dual-rail quantum bit*) allows for a simple error correction scheme; that is, for each of the pairs {a,b} and {c,d}, the only permissible states are $|01\rangle$ and $|10\rangle$. The states $|00\rangle$ and $|11\rangle$ correspond to loosing or gaining a photon, which will only happen when an error occurs. Thus, if we reject all such illegal results, we find that the error probability is

$$P_{\rm EC} = \frac{1}{2}\left[1 - \operatorname{sech}\frac{\gamma}{2}\right]\,, \tag{29}$$

that is, just the relative probability of finding $|0101\rangle$ (the wrong answer) to $|0110\rangle$ (the right answer). The improvement in error probability given by use of this simple-minded qubit error correction scheme is shown in Figure 5.

Even more interesting is what happens when the loss is *balanced* such that all four modes suffer identically. That is, we let $\$'_{F_\gamma} = B^\dagger \$^a_\gamma \$^b_\gamma \$^c_\gamma \$^d_\gamma K B$, as shown in Figure 4. Using this in U_1, we find that the diagonal elements of the final density matrix are

$$\begin{aligned}
{\rho'}^{\rm diag}_5 = {} & e^{-4\gamma}|0110\rangle\langle 0110| + \left(1 + e^{-4\gamma} - 2e^{-2\gamma}\right)\,|0000\rangle\langle 0000| \\
& + \frac{e^{-2\gamma} - e^{-4\gamma}}{2}\Big[|0001\rangle\langle 0001| + |0010\rangle\langle 0010| \\
& \qquad + |0100\rangle\langle 0100| + |1000\rangle\langle 1000|\Big]\,.
\end{aligned} \tag{30}$$

Furthermore, since the only legal state which can be obtained from the above is $|0110\rangle$ (there must be two photons in the output), we find that the after error correction, the error probability is *zero*. Physically, this occurs because of the symmetry of the damping. In classical optics, it is well-known that by balancing the loss in

an interferometer, unit visibility can be obtained. Analogously, for a single-photon interferometer (when only one photon is present in both arms), either the photon is lost (in which case the output is $|00\rangle$), or coherence is preserved perfectly. This behavior is the basis for the regenerative properties of the dual-rail quantum bit [13].

IV. DECOHERENCE IN THE SIMPLE QUANTUM COMPUTER

The above calculation indicates that errors due to loss can be prevented by using the appropriate design. However, the effect of phase damping is more insidious. To see this, let us substitute our results for the noisy Fredkin gate $\$_{F_\lambda}$ into U_0 and U_1 and calculate the output state ρ_5, just as before. Using Eqs.(17-21) and averaging over ϵ, we find that for the $k_1 = 0$ configuration the diagonal elements of the output density matrix are

$$\rho_5^{\rm diag}(k_1 = 0) = {\rm diag}\left[U'_0\,|0101\rangle\langle 0101|\,{U'}_0^{\dagger}\right] \tag{31}$$

$$= \frac{1+e^{-2\lambda}}{2}\,|0101\rangle\langle 0101| + \frac{1-e^{-2\lambda}}{2}\,|1001\rangle\langle 1001|\,, \tag{32}$$

Note that λ parameterizes the amount of decoherence, and for large λ, the two states $|0101\rangle\langle 0101|$ and $|1001\rangle\langle 1001|$ are equally probable. This mixed state results because decoherence in the Kerr media performs a partial "which path" measurement on the interferometer formed by modes a and b in the Fredkin gate.

On the other hand, when $k_1 = 1$, we use U'_1 to find the final result,

$$\rho_5^{\rm diag}(k_1 = 1) = {\rm diag}\left[U'_1\,|0101\rangle\langle 0101|\,{U'}_1^{\dagger}\right] \tag{33}$$

$$= \frac{(1-e^{-2\lambda})}{4}\,|0101\rangle\langle 0101| + \frac{(1-e^{-2\lambda})}{4}\,|1010\rangle\langle 1010| + \frac{(1+3e^{-2\lambda})}{4}\,|0110\rangle\langle 0110| + \frac{(1-e^{-2\lambda})^2}{4}\,|1001\rangle\langle 1001|\,. \tag{34}$$

In the limit of large λ, the four states $|0101\rangle\langle 0101|$, $|1010\rangle\langle 1010|$, $|0110\rangle\langle 0110|$ and $|1001\rangle\langle 1001|$ are equally probable. This means that our simple-minded error correction scheme (i.e. simply rejecting illegal states) fails! However, we have a more sophisticated method at our disposal.

If we have *a priori* knowledge that under perfect operation, the state $|\phi\rangle$ after the first Fredkin gate will be either $[|0101\rangle + |0110\rangle]/\sqrt{2}$ or $[|0101\rangle + |1010\rangle]/\sqrt{2}$, then we know that the space of legal results is spanned by $|\psi_0\rangle = [|0101\rangle + |1010\rangle]/\sqrt{2}$ and $|\psi_1\rangle = [|0101\rangle + 2|0110\rangle - |1010\rangle]/\sqrt{6}$. We may thus detect errors by measuring the component of $|\phi\rangle$ perpendicular to the $\{|\psi_0\rangle, |\psi_1\rangle\}$ space. The quantum circuit to do this is straightforward to design; basically, we perform a unitary transform to get $|\phi'\rangle = U\,|\phi\rangle$ which is either $|0101\rangle$ or $|1001\rangle$. When the last two labels are measured to be other than $|01\rangle$ we know an error has occurred, and the trial is rejected. Otherwise, we perform the inverse transform to restore the state, and continue as before. Using this scheme, we find that the probability of error in the final result decreases from $\lambda - \lambda^2$ to $11\lambda/18 - 47\lambda^2/162$ for small λ. The results are plotted in Figure 6.

The improvement achieved by the above indicates the possibility of using projective techniques to correct for phase randomization. Ideally, it would be nice to be able to detect and correct for errors due to decoherence, just as is possible for amplitude damping using dual-rail qubits. Along these lines, we have recently discovered a qubit representation which shows significant resistance to phase randomization [14]; instead of decohering at rate λ, we can achieve $4\lambda/N$ for arbitrary N by introducing $N-1$ ancilla qubits (which also decohere!) appropriately entangled.

V. CONCLUSION

Our analysis of the effects of loss and decoherence on a simple quantum computer indicates that although decoherence is a significant impediment to the realization of quantum computers, techniques exist which may be utilized to mitigate errors. In particular, the dual-rail quantum bit representation may be used to perfectly detect and correct errors due to amplitude damping. Other representations also exist which may be used against phase randomization [14,15].

Alternatively, new technologies may be developed which allow single photon qubits to interact without decoherence; the model used here is based on our understanding of bulk nonlinear optical materials, but other possibilities exist for resonant interactions which should have high $\chi^{(3)}$, negligible loss, and short interaction lengths. These are based on single-photonics technologies [16] that take advantage of our ability to engineer semiconductor devices. For example, we may envision a system consisting of the transmission of single photon dual-rail qubits over a fiber-optic link, with coding, decoding, and regeneration using exciton-polariton quantum logic gates and high-efficiency single-photon detectors.

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FIG. 1. The Chuang-Yamamoto quantum computer used to solve the one-bit Deutsch problem [7]. The apparatus in the dashed box is used by Bob to calculate $f_k(x)$, and everything else belongs to Alice. k_0 and k_1 control classical switches. Computation flows from left to right.

FIG. 2. A quantum-optical Fredkin gate constructed using a nonlinear Mach-Zehnder interferometer and cross-phase modulation via the Kerr interaction. The beamsplitter on the left (right) is described by B ($B^\dagger$).

FIG. 3. Classical transform functions for the 50/50 beamsplitter which are consistent Eq.(1). Note that $BaB^\dagger = (a-b)/\sqrt{2}$, and $BbB^\dagger = (a+b)/\sqrt{2}$.

FIG. 4. Model of quantum Fredkin gate with equal loss in all four modes.

FIG. 5. Error probability for the final measurement result in the $k_1k_0 = 10$ case, with and without error correction (lower and upper curves). For small γ, the improvement is substantial; $P_{\rm NOEC} \sim \gamma/2$ and $P_{\rm EC} \sim \gamma^2/16$, where loss is $10\gamma \log_{10} e$ [dB].

FIG. 6. Error probability for the final measurement result using a projective phase decoherence error correction scheme. The amount of damping is $10\lambda \log_{10} e$ [dB].

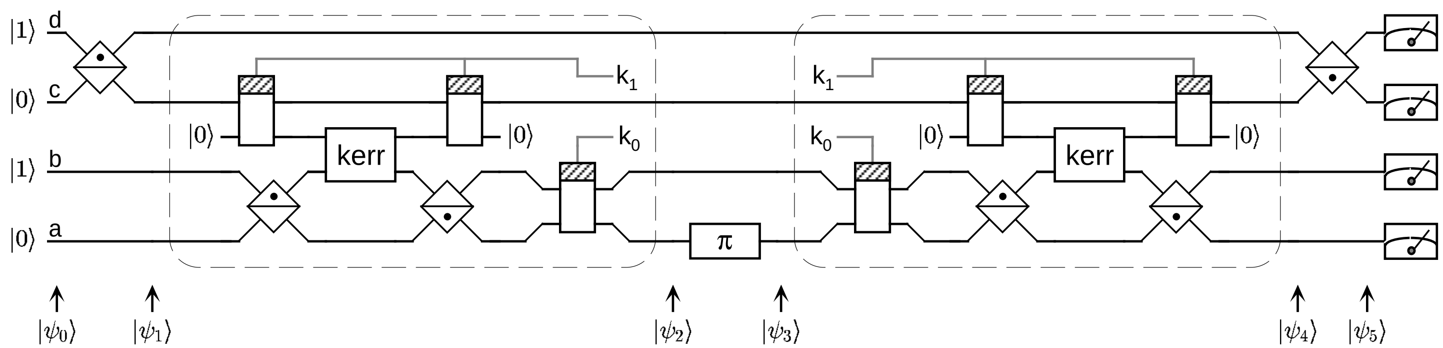

Figure 1

Chuang PRA qc+dec-12dec95

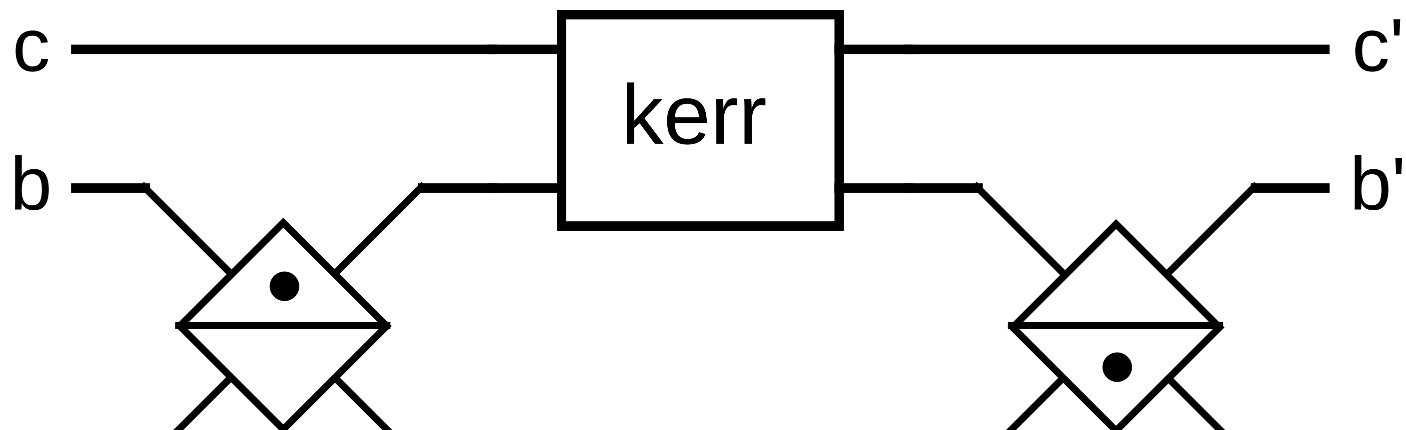

Figure 2

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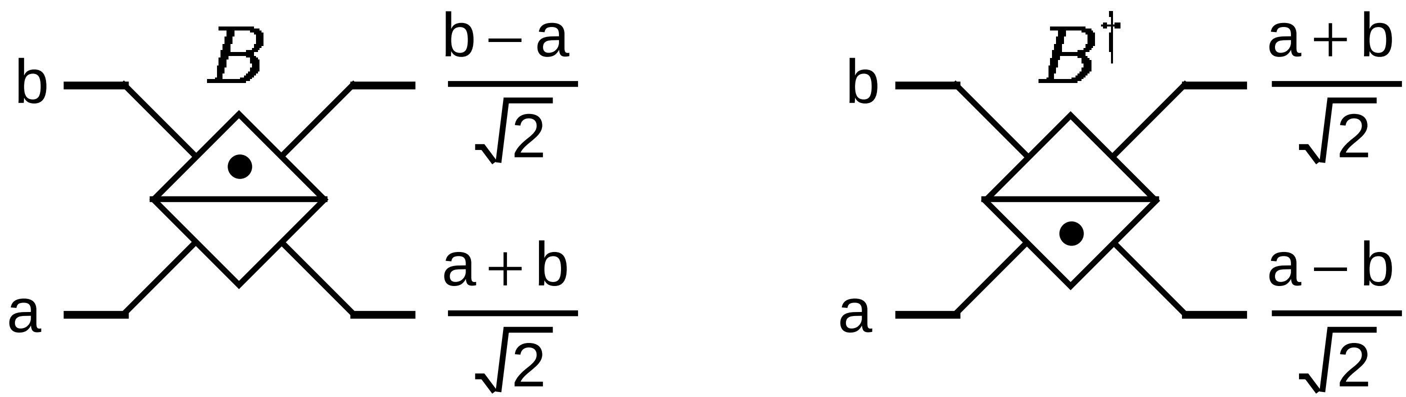

Figure 3

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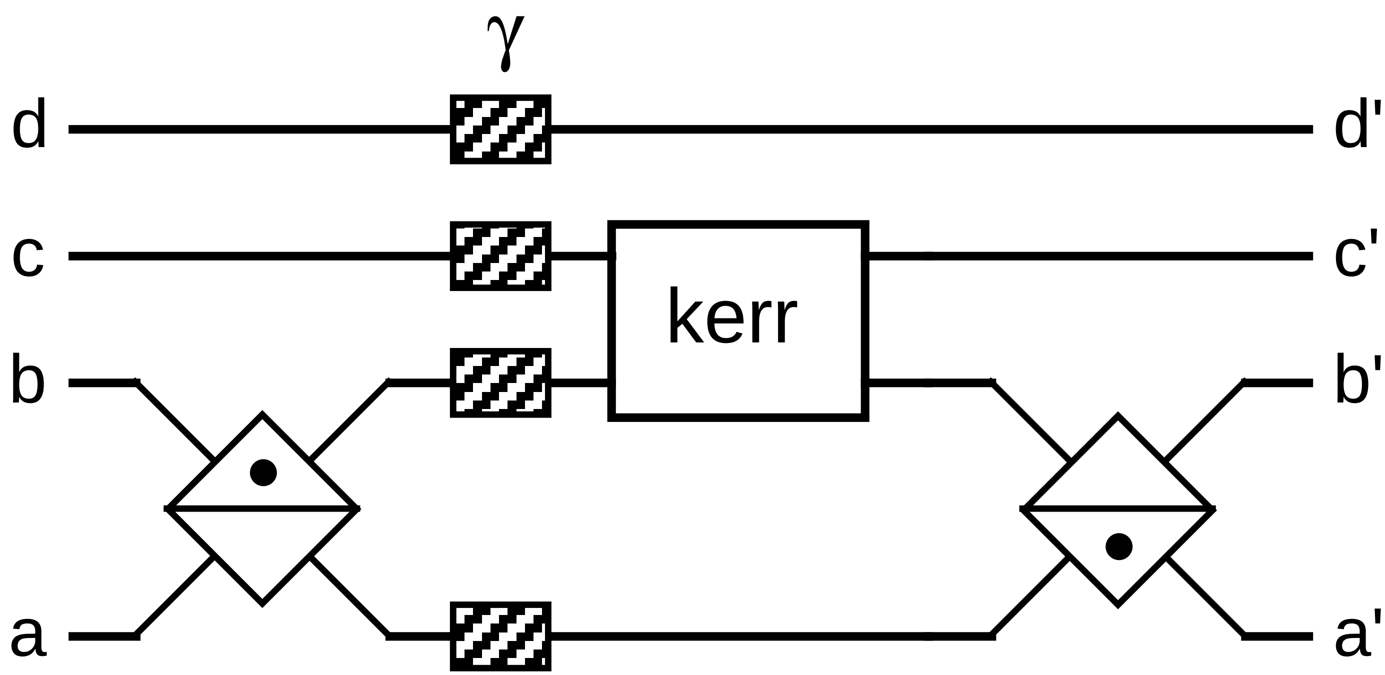

Figure 4

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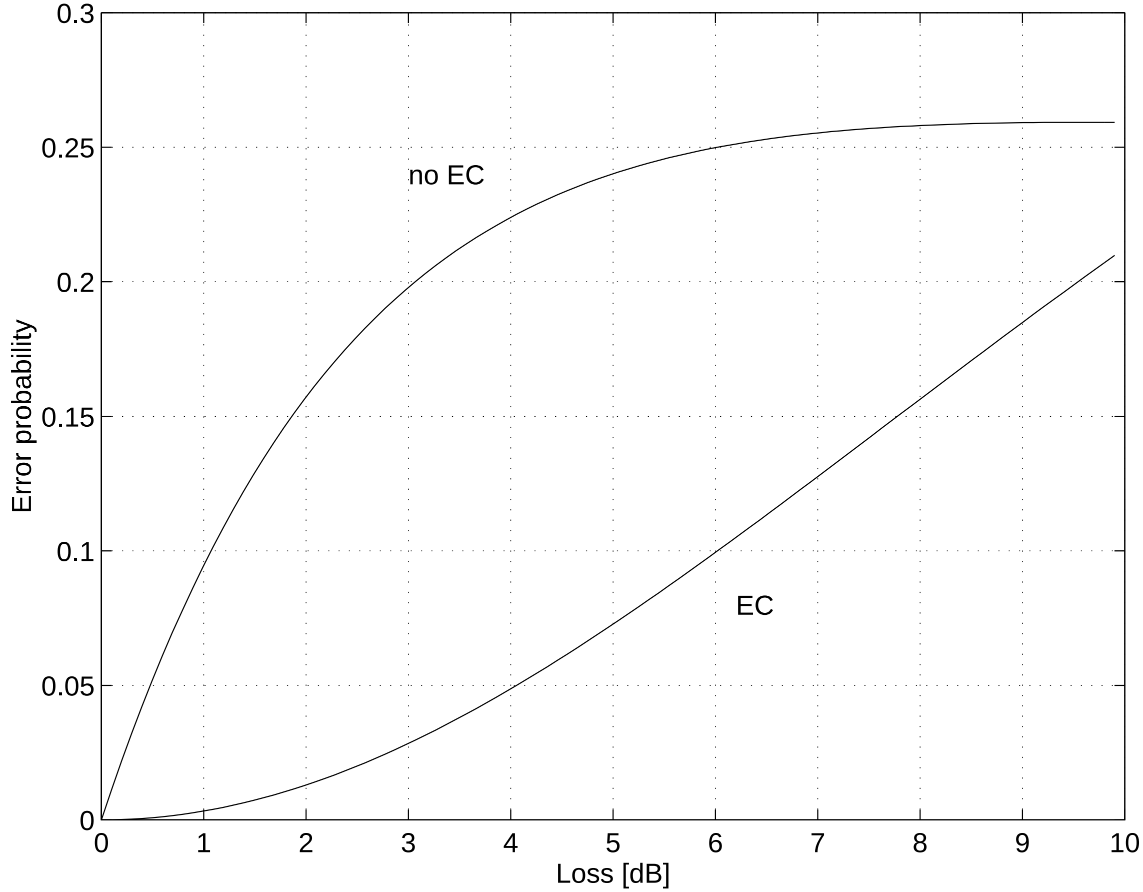

Figure 5

Chuang PRA qc+dec-12dec95

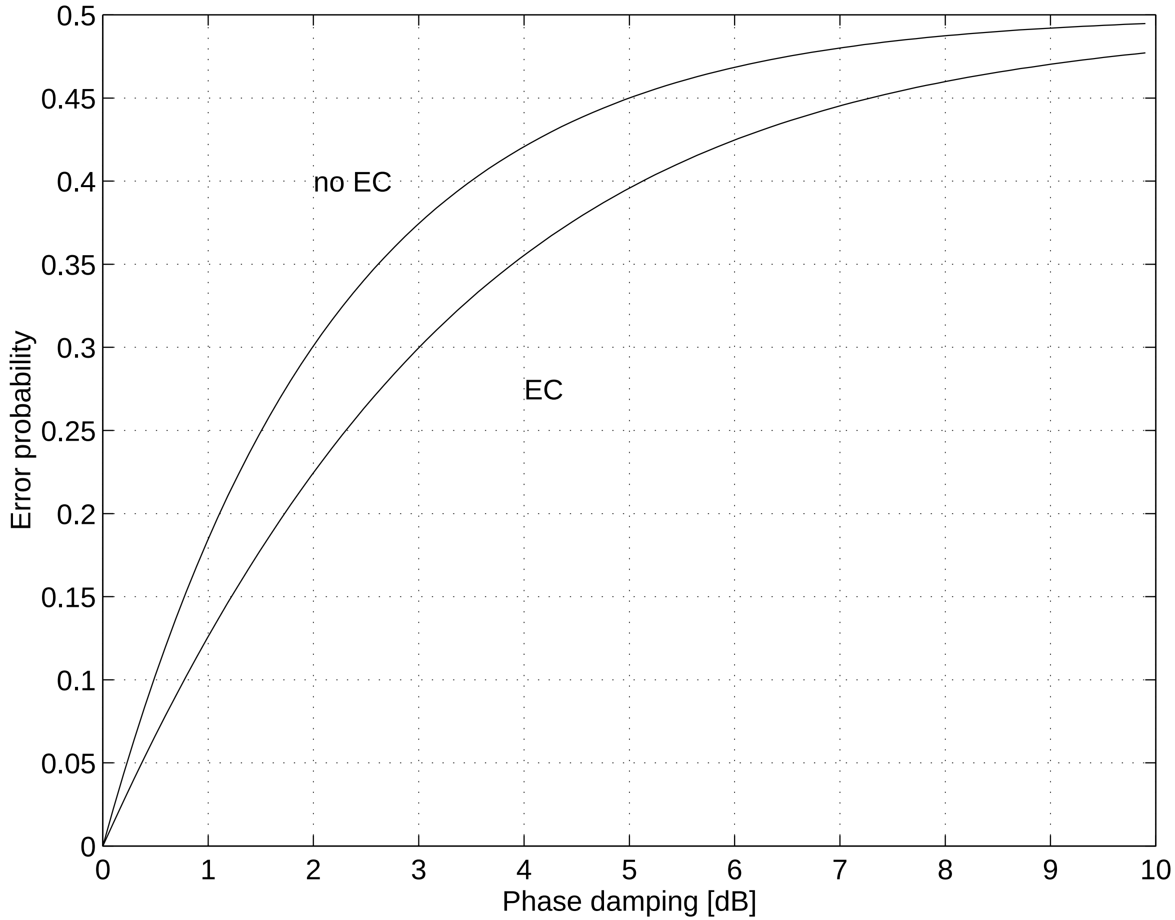

Figure 6

Chuang PRA qc+dec-12dec95